Pairwise Entanglement and Local Polarization of Heisenberg Model

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The pairwise entanglement and local polarization of the ground state are discussed by studying the Heisenberg XX model in finite qubit case. The results show that: the ground state is composed by the micro state with the minimal total spin 0 (for even qubit) or $\frac{1}{2}$ (for odd qubit), local polarization (LP) has intimate relation with the probability of the micro state in the ground state, the stronger the LP the smaller the probability, the same LP corresponding to the same probability; the pairwise entanglement of the ground state is the biggest in all the eigenvectors. We find when the qubit is small, the degenerate of state will decrease the pairwise entanglement, there has great different between the odd and the even qubit chain; when the qubit number is big, the effect of qubit number to the pairwise entanglement will disappear, the limited value will be round about 0.3424.

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Quantum entanglement play an important role in modern physics, besides using to test some fundamental questions of the quantum mechanics and using in quantum information processing, it can be used in the sensitivity of interferometric measurements such as quantum lithography [1], quantum optical gyroscope [2], quantum clock synchronization and positioning [3] and frequency metrology [4], it also play a central role in the study of strongly correlated quantum systems [5], especially, ground-state entanglement has some connection with the quantum phase transition [6], Mott insulator-superfluid transition and quantum magnet-paramagnet transition. Those usability are all based on the production of entanglement, which is dependent on our recognition to it, because we know the two level system and their interaction are the necessary condition for the pairwise entanglement, so photons [7], trapped ions [8] (or trapped atoms [9], crystal lattices [10], Josephson junctions [11] and Bose-Einstein condensates [12], which have two level system and interaction, are often used to produce entanglement. Looking for the law of entanglement become one of the most important thing.

Spin chain is a nature candidate for producing pairwise entanglement, it has been used to construct a quantum computer [13], C-NOT gate [14] and swap gate [15]; it can also be used to quantum communication [16, 17, 18]. For the pairwise entanglement in spin chain, the relevant work are around how to control and mike it maximal, the effective factors are temperature, interchange coupling [15, 19, 20], magnetic field and system impurity [21, 22, 23], some useful conclusions are concluded for the few qubits cases. Besides those factors, the length of chain is also an effective factor to the pairwise entanglement, the previous work in this point are some special cases: for $|W_N>$ state, the concurrences between any two qubits are all equal to $2/N$ [24, 25]; Koashi *et al.* show [26] that the maximum degree of entanglement between any pair of qubits of a N-qubit symmetric state is $2/N$; Connor *et al.* [27] calculated the pairwise entanglement in spin chain which satisfied two conditions: the state $|\psi>$ of the ring is an eigenvector of the total z-component of spin; neighboring particles cannot both be in the state $|\uparrow>$. Those work are contribute to our understanding of the properties of entanglement and its distributed among many objects, but even the work about pairwise entanglement is far from completeness, for example, how the macro quality, such as total spin , LP and the length of chain, effect the pairwise entanglement?

In this paper, we will use the simplest spin model (Heisenberg XX model) to study how the total spin, LP and the length of chain effect the pairwise entanglement. The results can help us to deep understand the properties of entanglement and give some useful guidance to the experiments in solid system.

No one knows if nature only needs pairwise entanglement, but it is no doubt that it is very useful in quantum information processing, and it is necessary to find all its properties. Before beginning, let us give a brief review of the measurement of the pairwise entanglement and the notation in this paper: the concept of entanglement of formation and concurrence [28, 29]. Concurrence C range from zero to one and it is monotonically relate to entanglement of formation, so that concurrence C is a kind of measure of entanglement. ρ is the density matrix of system, $\rho_{12} = Tr_{non(12)}\rho$ (mixed or pure) is the reduced density matrix of the pair (the nearest or the non-nearest), where $1, 2$ mean the first qubit and the second qubit in the Heisenberg XX ring. The concurrence corresponding to the density matrix is defined as $C_{12} = \max\{\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4, 0\}$, where λ_k, $k = 1, 2, 3, 4$ are the square roots of the eigenvalues of the operator $\hat{\rho}_{12} = \rho_{12}(\sigma_1^y \otimes \sigma_2^y)\rho_{12}^*(\sigma_1^y \otimes \sigma_2^y)$ in descending order.

The Hamiltonian of $S=\frac{1}{2}$ Heisenberg XX chain is $H_{xx}=\sum_{n=1}^{N}2J(S_n^xS_{n+1}^x+S_n^yS_{n+1}^y)$, where $S_{N+1}=S_1$, $J<0$ and $J>0$ are corresponding to ferromagnetic and anti-ferromagnetic area. Using the relations $S^\alpha=\sigma^\alpha/2$ $(\alpha=x,y)$ and $\sigma^\pm=(\sigma^x\pm i\sigma^y)/2$, the system Hamiltonian can be rewritten as $H=J\sum_{n=1}^{N}(\sigma_n^+\sigma_{n+1}^-+\sigma_{n+1}^+\sigma_n^-)$.

The eigenvalues and eigenvectors.- The most important step of calculating concurrence C_{12} is to get the eigenvalues and eigenvectors of the system. Usually, suppose $|\psi>=\sum_{i=0}^{2^N-1}a_i|i>$ and using $H|\psi>=E|\psi>$ we can get all eigenvalues and eigenvectors of the system, then pick out the ground state in them. For N-qubit case, we must handle a matrix with order 2^N, the calculation will be very difficult when N is large.

In order to overcome the difficulty of calculation, we find a way to reduce the coefficients in the eigenvectors. As we know, in four and five qubit case, the ground sates of the ferromagnetic and anti-ferromagnetic area are all composed by the micro state which has minimal total spin $\frac{1}{2}$ (for odd qubit) or 0 (for even qubit); the micro states, which have the same macro state, have the same probability. So we generalize this to more qubit case, the results show that this method is still effective. Details are shown in the following:

For three qubit chain, the ground state is composed by $|j>$, or $|j,j+1>$, where $j=1,2,3$ is a space coordinate and at this site the spin is up, the others are spin down. All the three $|j>$ has the same probability, so we can write the eigenvector as $|\psi(m)>=C_1\sum_{j=1}^{3}e^{i\frac{2j\pi}{3}m}|j>$, where $m=1,2,3$, C_1 satisfy $H|\psi>=E|\psi>$ and $3|C_1|^2=1$. In the ferromagnetic area $(J<0)$, the eigenvalue is $E_3^+=2J$, the corresponding eigenvectors(two degenerate) are $|\psi(3)>_1^+=\frac{1}{\sqrt{3}}\sum_{j=1}^{3}|j>$, $|\psi(3)>_2^+=\frac{1}{\sqrt{3}}\sum_{j=1}^{3}|j,j+1>$. In the anti-ferromagnetic area $(J>0)$, the eigenvalue is $E_3^-=-J$, the corresponding eigenvectors(four degenerate) are $|\psi(1)>_1^-=\frac{1}{\sqrt{3}}\sum_{j=1}^{3}e^{i\frac{2j\pi}{3}}|j>$, $|\psi(2)>_2^-=\frac{1}{\sqrt{3}}\sum_{j=1}^{3}e^{i\frac{2j\pi}{3}2}|j>$, $|\psi(1)>_3^-=\frac{1}{\sqrt{3}}\sum_{j=1}^{3}e^{i\frac{2j\pi}{3}}|j,j+1>$, $|\psi(2)>_4^-=\frac{1}{\sqrt{3}}\sum_{j=1}^{3}e^{i\frac{2j\pi}{3}2}|j,j+1>$.

For four qubit chain, the ground state is composed by $|j,j+1>$ (i.e.the nearest two neighbours spin up) and $|j,j+2>$ (the next nearest two neighbours spin up), where $j=1,2,3,4$. We write the eigenvector of four qubit case as $|\psi(m_i)>=C_1\sum_{j=1}^{4}e^{i\frac{2j\pi}{4}m_1}|j,j+1>$ $+C_2\sum_{j=1}^{2}e^{i\frac{2j\pi}{2}m_2}|j,j+2>$, where $m_1=1,2,3,4,m_2=1,2$, C_1,C_2 satisfy $H|\psi>=E|\psi>$ and $4|C_1|^2+2|C_2|^2=1$. The eigenvalues in both magnetic area are $E_4^+=2\sqrt{2}J$ $(J<0)$, $E_4^-=-2\sqrt{2}J$ $(J>0)$, the corresponding eigenvector has no degenerate in both magnetic areas, they are $|\psi(4,2)>^+=\frac{1}{2\sqrt{2}}\sum_{j=1}^{4}|j,j+1>$ $+\frac{1}{2}\sum_{j=1}^{2}|j,j+2>$, $|\psi(4,2)>^-=\frac{1}{2\sqrt{2}}\sum_{j=1}^{4}|j,j+1>$ $-\frac{1}{2}\sum_{j=1}^{2}|j,j+2>$.

For five qubit chain, the ground state is composed by $|j,j+1>$ and $|j,j+2>$, or $|j,j+1,j+2>$ and $|j,j+2,j+3>$, where $j=1,2,3,4,5$. The eigenvector is $|\psi(m_i)>=C_1\sum_{j=1}^{5}e^{i\frac{2j\pi}{5}m_1}|j,j+1>+C_2\sum_{j=1}^{5}e^{i\frac{2j\pi}{5}m_2}|j,j+2>$, where $m_1,m_2(=1,2,3,4,5)$, C_1,C_2 satisfy $H|\psi>=E|\psi>$ and $5|C_1|^2+5|C_2|^2=1$. In the ferromagnetic area, the eigenvalue is $E_5^+=(\sqrt{5}+1)J$, the corresponding eigenvectors(two degenerate) are $|\psi(5,5)>_1^+=0.235|j,j+1>+0.380|j,j+2>$, $|\psi(5,5)>_2^+=0.235|j,j+1,j+2>+0.380|j,j+2,j+3>$, the eigenvalue is $E_5^-=-\frac{1}{2}(3+\sqrt{5})J$, the corresponding eigenvectors(four degenerate) are $|\psi(1,1)>_1^-=C_{11}\sum_{j=1}^{5}e^{i\frac{2j\pi}{5}}|j,j+1>+C_{12}\sum_{j=1}^{5}e^{i\frac{2j\pi}{5}}|j,j+2>$, $|\psi(4,4)>_2^-=C_{21}\sum_{j=1}^{5}e^{i\frac{2j\pi}{5}4}|j,j+1>+C_{22}\sum_{j=1}^{5}e^{i\frac{2j\pi}{5}4}|j,j+2>$, $|\psi(1,1)>_3^-=C_{31}\sum_{j=1}^{5}e^{i\frac{2j\pi}{5}}|j,j+1,j+2>+C_{32}\sum_{j=1}^{5}e^{i\frac{2j\pi}{5}}|j,j+2,j+3>$, $|\psi(4,4)>_4^-=C_{41}\sum_{j=1}^{5}e^{i\frac{2j\pi}{5}4}|j,j+1,j+2>+C_{42}\sum_{j=1}^{5}e^{i\frac{2j\pi}{5}4}|j,j+2,j+3>$. where parameters C_{ij} are: $C_{11}=-0.190+0.138i$, $C_{21}=-0.190-0.138i$, $C_{31}=0.073+0.224i$, $C_{41}=0.073-0.224i$, $C_{12}=C_{22}=C_{32}=C_{42}=0.380$.

For six qubit chain, the ground state is composed by $|j,j+1,j+2>$, $|j,j+2,j+3>$, $|j,j+3,j+4>$ and $|j,j+2,j+4>$, where $j=1,2,3,4,5,6$, the eigenvector is $|\psi(m_i)>=C_1\sum_{j=1}^{6}e^{i\frac{2j\pi}{6}m_1}|j,j+1,j+2>+C_2\sum_{j=1}^{6}e^{i\frac{2j\pi}{6}m_2}|j,j+2,j+3>+C_3\sum_{j=1}^{6}e^{i\frac{2j\pi}{6}m_3}|j,j+3,j+4>$ $,+C_4\sum_{j=1}^{2}e^{i\frac{2j\pi}{2}m_4}|j,j+2,j+4>$ where $m_{1,2,3}=1,2,3,4,5,6$,$m_4=1,2$, C_1,C_2,C_3,C_4 are determined from $H|\psi>=E|\psi>$ and $6|C_1|^2+6|C_2|^2+6|C_3|^2+2|C_4|^2=1$. The eigenvalues in both magnetic area are $E_6^+=4J$ $(J<0)$, $E_6^-=-4J$ $(J>0)$, the corresponding eigenvector has no degenerate in both magnetic areas, they are $|\psi(6,6,6,2)>^+=\frac{1}{6\sqrt{2}}\sum_{j=1}^{6}|j,j+1,j+2>+\frac{1}{3\sqrt{2}}\sum_{j=1}^{6}|j,j+2,j+3>$ $+\frac{1}{3\sqrt{2}}\sum_{j=1}^{6}|j,j+3,j+4>+\frac{1}{2\sqrt{2}}\sum_{j=1}^{2}|j,j+2,j+4>$; $|\psi(3,3,3,1)>^-=\frac{1}{6\sqrt{2}}\sum_{j=1}^{6}e^{i\frac{2j\pi}{6}3}|j,j+1,j+2>$ $+\frac{1}{3\sqrt{2}}\sum_{j=1}^{6}e^{i\frac{2j\pi}{6}3}|j,j+2,j+3>+\frac{1}{3\sqrt{2}}\sum_{j=1}^{6}e^{i\frac{2j\pi}{6}3}|j,j+3,j+4>+\frac{1}{2\sqrt{2}}\sum_{j=1}^{2}e^{i\frac{2j\pi}{2}}|j,j+2,j+4>$.

For more than six qubit chain, we calculated the case of seven and eight qubit, the eigenvalue and eigenvectors of seven qubit in ferromagnetic (anti-ferromagnetic is omitted) area are: $E_7=4.4611J$, $|\psi>_1=\sum_{kl}(C_{kl}\sum_{j=1}^{7}|j,j+k,j+l>)$, $|\psi>_2=\sum_{kl}(C_{kl}\sum_{j=1}^{7}|\bar{j},\overline{j+k},\overline{j+l}>)$, where $kl=12,13,14,15,24$ is composed number, $C_{12,13,14,15,24}=0.064,0.143,0.178,0.143,0.257$, $\bar{j}$ means the spin in this site is down. The eigenvalue and eigenvectors of eight qubit are: $E_8=2\sqrt{4+2\sqrt{2}}J,|\psi>=$

$\sum_{kln}(C_{kln}\sum_{j=1}^{8}|j,j+k,j+l,j+n>)+C_{145}\sum_{j=1}^{4}|j,j+1,j+4,j+5>+C_{246}\sum_{j=1}^{2}|j,j+2,j+4,j+6>$, where $kln = 123,124,125,126,134,135,136,146$ is also composed number, $C_{123,124,125,126,134,135,136,146} = 0.022,0.056,0.074,0.056,0.074,0.136,0.127,0.136, C_{145} = 0.147, C_{246} = 0.417$. Using this ways the ground state of N qubit can be calculated as long as we can manufacture a matrix of order $\frac{(N-1)!}{[\frac{N}{2}]!(N-[\frac{N}{2}])!}$, in fact only the case of $N \leq 15$ (the matrix order of $N = 15$ is 429, great less than $2^{15} = 32768$) can be calculate. However, $N = 15$ is large for the quantum dot or the Josephson junctions, which can realize the spin system.

The degenerate degree of the ground state is different for odd and even qubit chain: odd qubit chain has degenerate while even qubit case has not. For odd qubit case, different magnetic area has different degenerate, the degenerate in anti-ferromagnetic area is higher than ferromagnetic area. We can understand them from the following: (1) for the minimal total spin of even (N) qubit chain, they only have one possibility of $\frac{N}{2}$ spin up, of cause they have no degenerate; (2) for the minimal total spin of odd(N) qubit chain, they have two equal possibility, $\frac{N-1}{2}$ or $\frac{N+1}{2}$ spin up, so it at least has two degenerate, in the anti-ferromagnetic area, when two spin exchange, a minus will appear, so it has four degenerate. This degenerate will decrease the pairwise entanglement. The degenerate will decrease the pairwise entanglement.

Local Polarization (LP).- From the above eigenvectors, we can see a very important phenomena about LP. We summarized the micro state of even qubit into Table 1.

Table 1. The LP and probability of micro state, "P-Mic" is instead of the probability of the micro state

N	Micro state	P-Mic
4-qubit	$\|j,j+1>$	$\frac{1}{8}$
	$\|j,j+2>$	$\frac{1}{4}$
6-qubit	$\|j,j+1,j+2>$	$\frac{1}{72}$
	$\|j,j+2,j+3>$	$\frac{1}{18}$
	$\|j,j+3,j+4>$	$\frac{1}{18}$
	$\|j,j+2,j+4>$	$\frac{1}{8}$
8-qbit	$\|j,j+1,j+2,j+3>$	0.022^2
	$\|j,j+1,j+2,j+4>$	0.056^2
	$\|j,j+1,j+2,j+6>$	0.056^2
	$\|j,j+1,j+2,j+5>$	0.074^2
	$\|j,j+1,j+3,j+4>$	0.074^2
	$\|j,j+1,j+3,j+6>$	0.127^2
	$\|j,j+1,j+3,j+5>$	0.136^2
	$\|j,j+1,j+4,j+6>$	0.136^2
	$\|j,j+1,j+4,j+5>$	0.147^2
	$\|j,j+2,j+4,j+6>$	0.417^2

Now let us give an analysis of Table 1 in details. In four qubit, the components are $|j,j+1>$ and $|j,j+2>$, the former has big LP and small probability, while on the country for the later. The components of six qubit are $|j,j+1,j+2>, |j,j+1,j+3>, |j,j+1,j+4>$ and $|j,j+2,j+4>$, the corresponding probability of them are $\frac{1}{72}, \frac{1}{18}, \frac{1}{18}, \frac{1}{8}$, $|j,j+1,j+2>$ is the smallest, $|j,j+2,j+4>$ is the biggest, the middle two are equal. Obviously, the LP of them are very different, $|j,j+1,j+2>$'s is the strongest (with high degree of order), $|j,j+2,j+4>$'s is the weakest (with low degree of order), the LP of the middle two are equal (they have the same probability). In eight qubit, for simplicity we use the composed number kln instead of the micro state (for example, 123 instead of $|j,j+1,j+2,j+3>$), 123 has the strongest LP and the smallest probability, 246 has the weakest LP and the biggest probability; there have three same pairs in the middle, they are 124 and 126, 125 and 134, 135 and 146, they have the same LP and probability. The total spin of them (even qubit) are all equal zero. Similarly phenomena exists in odd qubit cases (total spin equal $\frac{1}{2}$).

That is to say, the LP and the probability of micro state have point to point relation: the stronger the LP the smaller the probability, the same LP corresponding to the same probability. The micro states with the biggest LP has no pairwise entanglement, the micro states with the smallest LP has the maximal pairwise entanglement between any two qubits. The other's are in the middle of two extremum. They compose the stablest state, no more or no less, in some degree, spin chain shows harmonic and variety, just like nature as.

The Pairwise Entanglement.- Using the standard concurrence theory, we can get the pairwise entanglement of the ground state as the following, see Table 2:

Table 2. The table of C_{12} at ground state, "f" and "a-f" are instead of ferromagnetic and anti-ferromagnetic area.

N		2-qubit	4-qubit	6-qubit	8-qubit
$\mathcal{C}_{12}$	f	1	0.45711	0.38889	0.37048
$\mathcal{C}_{12}$	$a-f$	1	0.45711	0.38889	0.37048
N		1-qubit	3-qubit	5-qubit	7-qubit
$\mathcal{C}_{12}$	f		0.33333	0.33666	0.33787
$\mathcal{C}_{12}$	$a-f$		0	0.21305	0

From Table 2, we find that C_{12} (the concurrence between the nearest qubits) is the same at different ferromagnetic area for even qubit chain, while different for odd qubit chain; for odd qubit chain, C_{12} at anti-ferromagnetic area is bigger than ferromagnetic area. These different come form the different of degenerate degree: even qubit chain has no degenerate while odd qubit chain has; for odd qubit chain, the degenerate degree at anti-ferromagnetic area is double times than that of ferromagnetic area. Those can be seen clearly from eigenvectors and can be testified by calculating C_{12} of any single state. The bigger the degenerate the smaller the entanglement, so decreasing the degenerate degree will increasing pairwise entanglement, introducing magnetic field is a proper ways to eliminate the degenerate and increase the entanglement, for example, if we eliminate the degenerate in the ground state of three-qubit case, we will get $C_{12} = 0.6667$ in the ferromagnetic area. The

pairwise entanglement in anti-ferromagnetic area of odd qubit chain seems complex, because there exist entanglement in 5-qubit chain while do not exist in 3-qubit and 7-qubit chain, it worth to be discussed further.

From Table 2, we see that C_{12} decreases with qubit number for even qubit chain while on the country for odd qubit chain. For understanding this more clearly, we plot the diagram of C_{12} with qubit number at ferromagnetic area, see Figure 1:

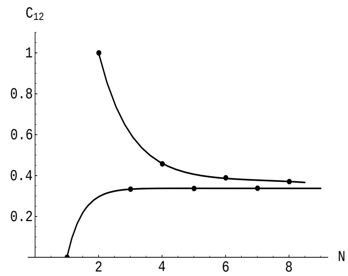

Figure 1. The diagram of C_{12} with qubit number at ferromagnetic area

Obviously, from the trend in the diagram of Figure 1, we may infer that there exist a limit value of C_{12} and it will be round about 0.3424, the trend in the anti-ferromagnetic area are similarly with ferromagnetic area as well as its limit value. That is to say, as the qubit number increase, the effect of qubit number to the pairwise entanglement will disappear, this is coincidence with the fact that single qubit will give smaller and smaller effect to the system as the qubit number increase. This conclusion tells us if we want to use such pairwise entanglement, a short chain is enough.

Up to now, we constructed the eigenvectors of the ground state of Heisenberg XX chain in finite case, discussed the local polarization (LP) and probability of the micro states and calculated the pairwise entanglement. There are three interesting results can help us to deep understand the properties of entanglement and give us some guidance in the future experiments.

(1) The micro states which compose the ground state have the minimal total spin and the same macro state, then they have the same probability. In finite qubit case, this way can take us great advantage for getting the eigenvalues and eigenvectors. We suppose that minimal total spin for a macro state is a necessary condition for the ground state, this point is coincidence with other law of nature: such as law of minimal energy for a stable system, law of minimal superficial area for a drop of liquid and law of minimal action for the real orbit of a subject *etc.*.

(2) The pairwise entanglement of the ground state is the maximal in all eigenvectors. Because pairwise entanglement is a kind of correlated quality, the more correlation between the subsystem, the more stability there exist in the system. So entanglement can tells us whether a state is the ground state or not, this is very interesting and maybe this property will became a criterion for the ground state, if that, entanglement will have a new usage.

(3) The LP and the probability of the micro state have intimate relation: the stronger the LP the smaller the probability, the same LP corresponding to the same probability. The state of the smallest LP is a special state, any qubit pair (nearest or non-nearest) has the maximal pairwise entanglement (concurrence=1). This chain is an ideal quantum entanglement channel, which can be used to teleport a quantum state and realize a quantum network. The important thing is how to design such a state in real system. If we can control a single spin with magnetic field then we can realize such an eigenvector.

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